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NanoSaclay Laboratoire d'Excellence en Nanosciences et Nanotechnologies
EDOM Ecole Doctorale Ondes et Matière
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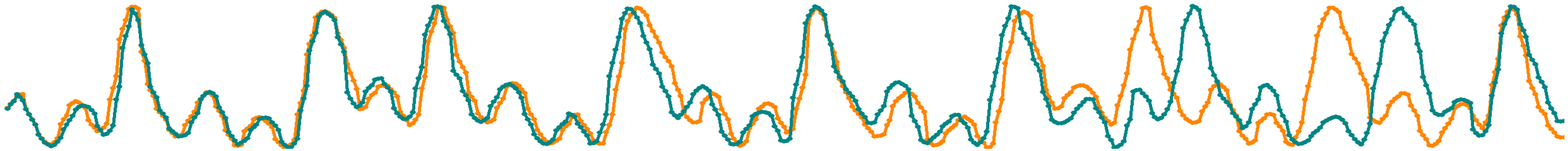
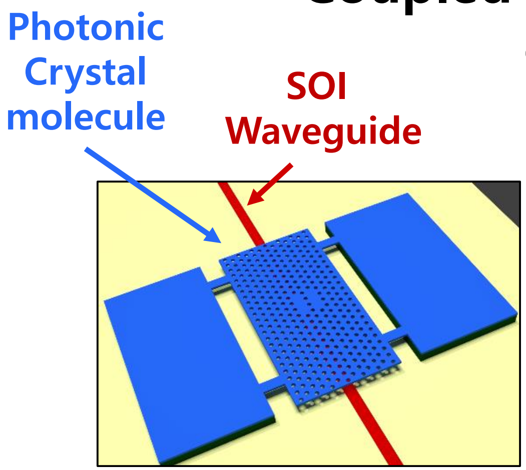
Nonlinear dynamics of coupled electromechanical & optomechanical resonators

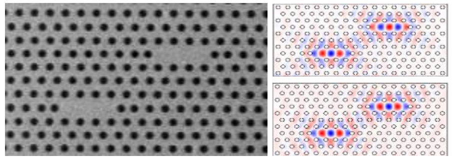
Guilhem Madiot, Franck Correia, Sylvain Barbay & Rémy Braive



Annual Labex NanoSaclay Meeting - Flagship Condor - 19/09/2019

Coupled optomechanical cavities





- Integrated nano-optomechanical platform
- Sustain MHz mechanical modes
- 2D photonic Crystal defect cavities

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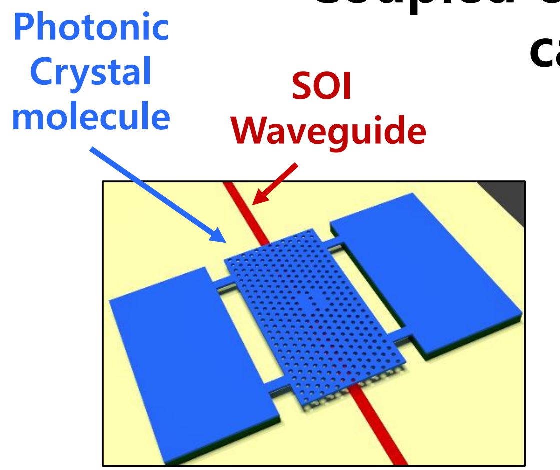

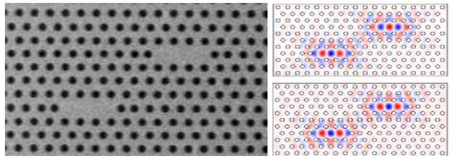

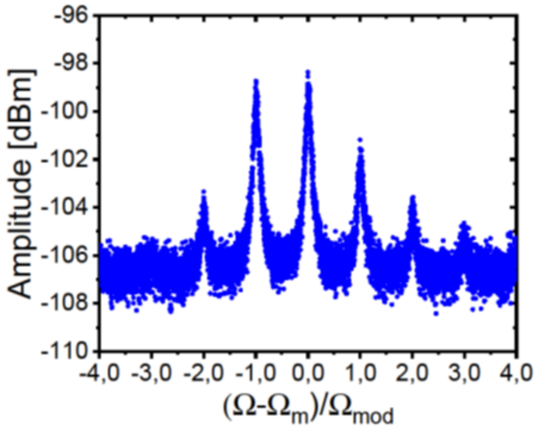

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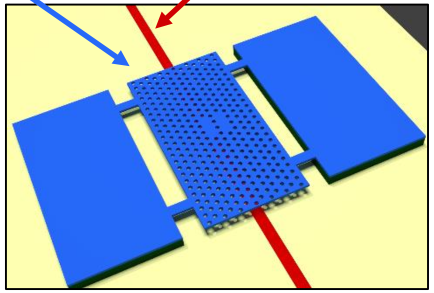
- ➢ Sidebands generation using electro-optical modulation
- ➢ Sideband enhancement through thermo-optical nonlinearity

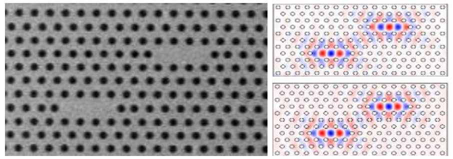
collab. theory : Karl Pelka, André Xuereb, Univ of Malta

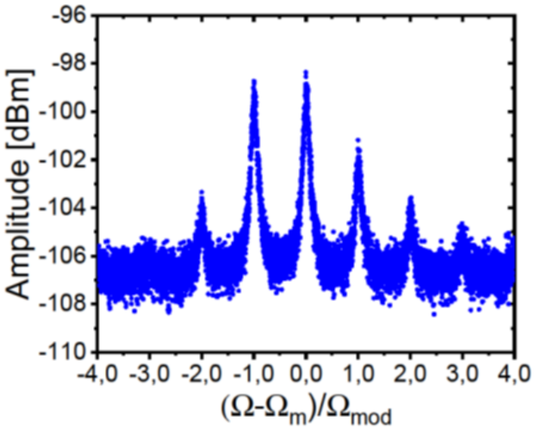
Coupled optomechanical cavities

Photonic Crystal molecule

SOI Waveguide







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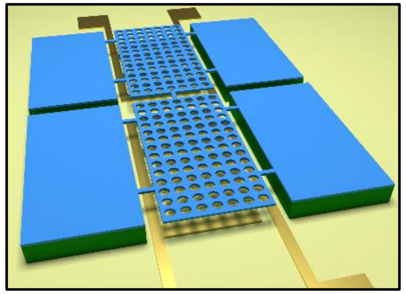
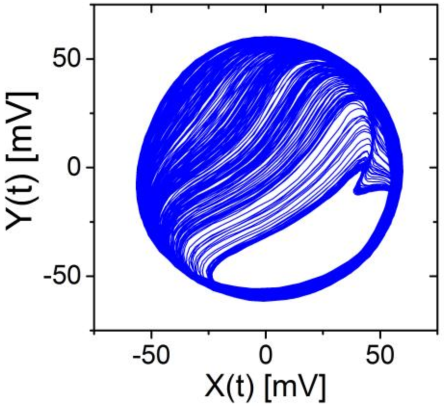
Coupled Electromechanical resonators



- Mechanically coupled NEMS



➢ Wave mixing sidebands generation

➢ **Classical chaos**

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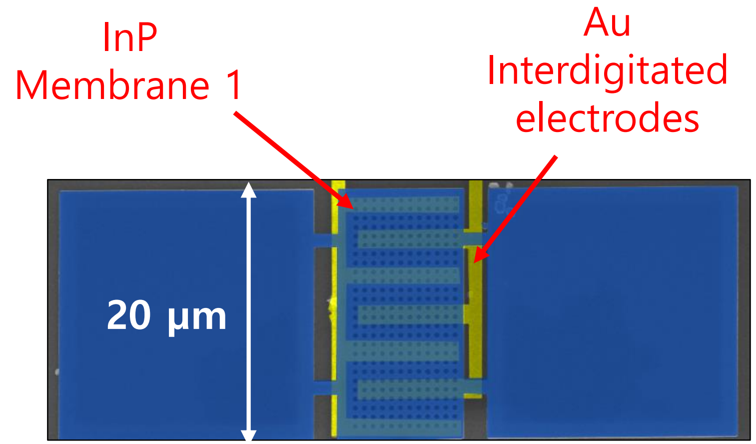
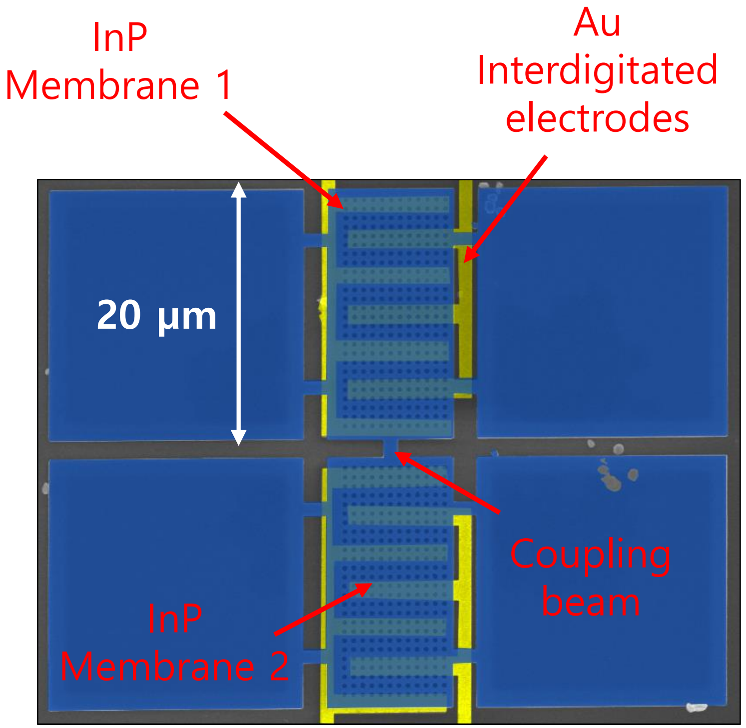
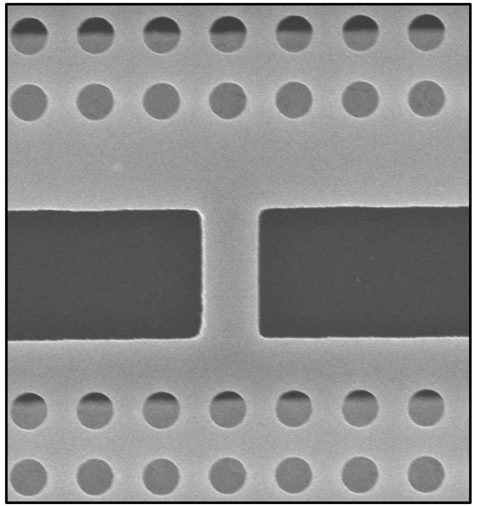
Coupled electromechanical nano-membranes



Individual properties
- Surface : $10x20\ \mu m^2$
- Thickness : 350 nm
- Mass : ~420 pg

Coupled electromechanical nano-membranes





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- Length : 1,5µm
- Width : ~1,0 µm

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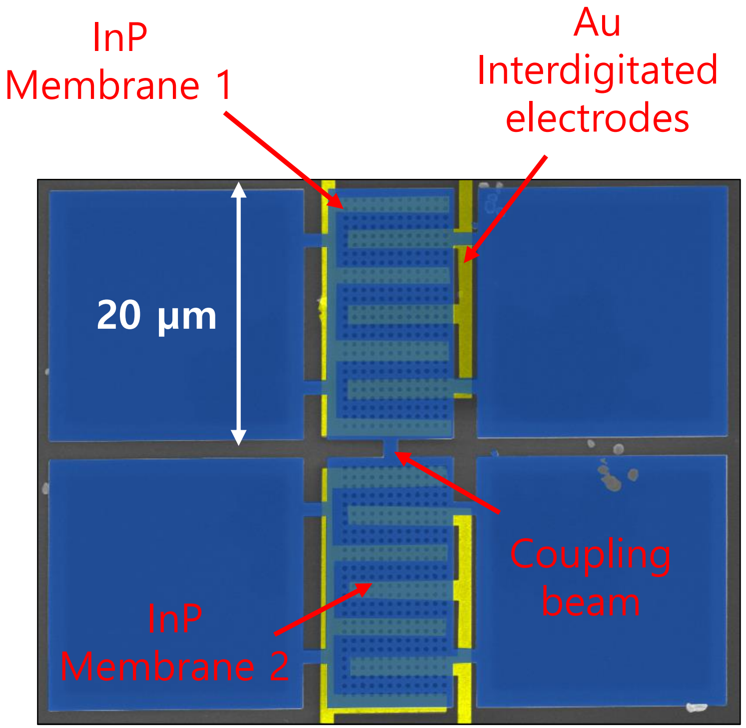

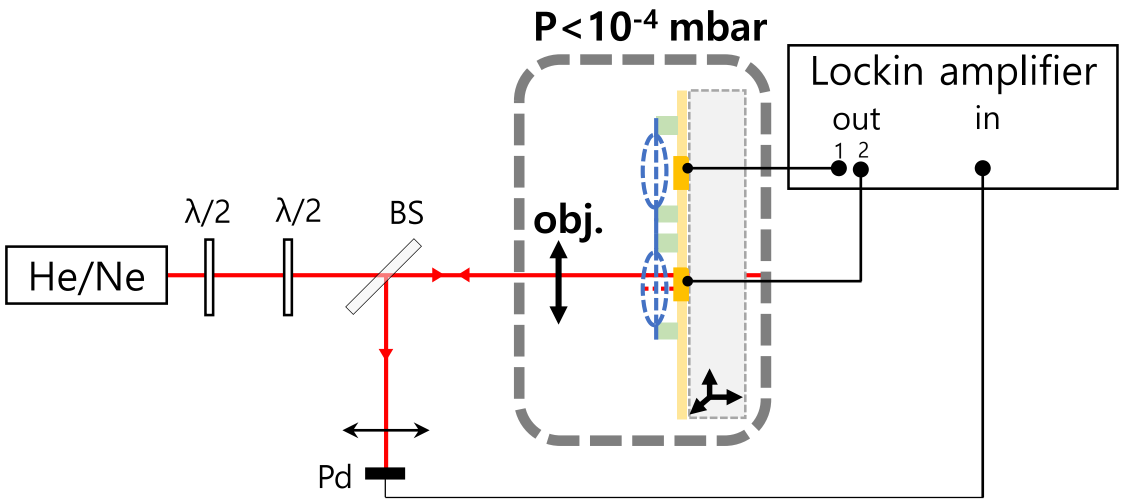

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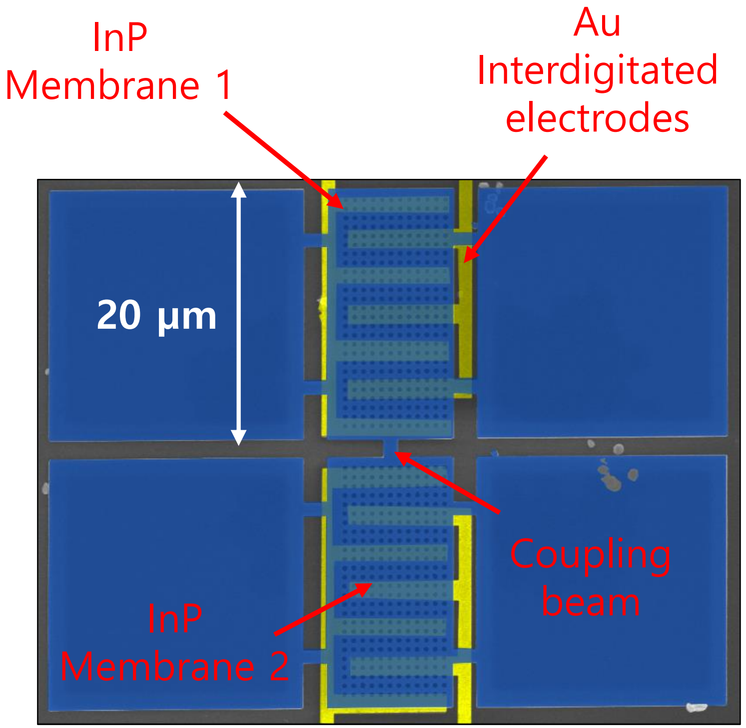

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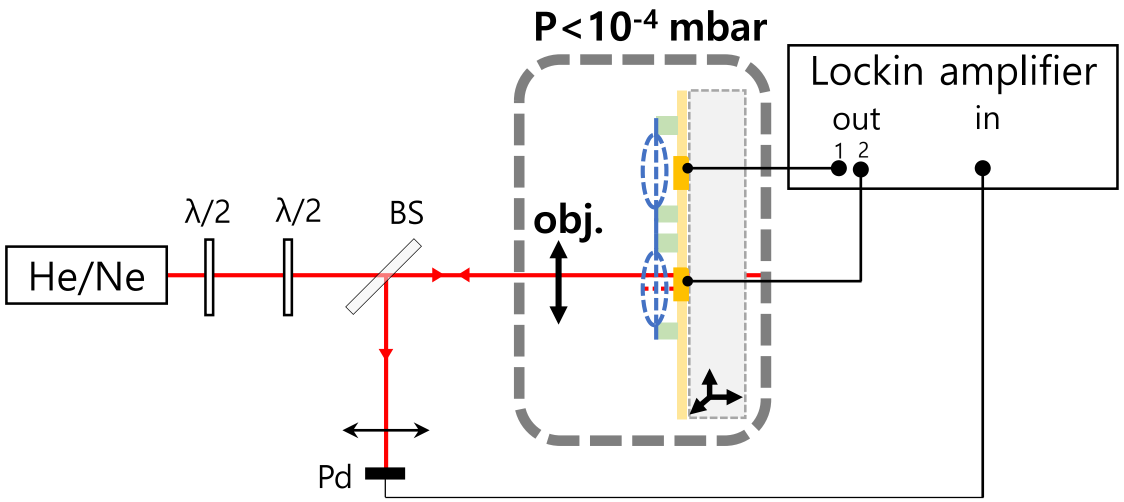
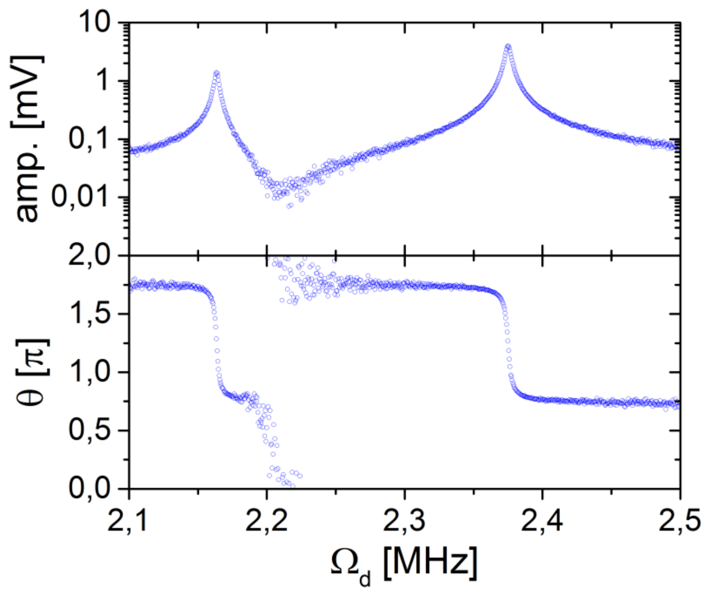


Lockin amplifier outputs :

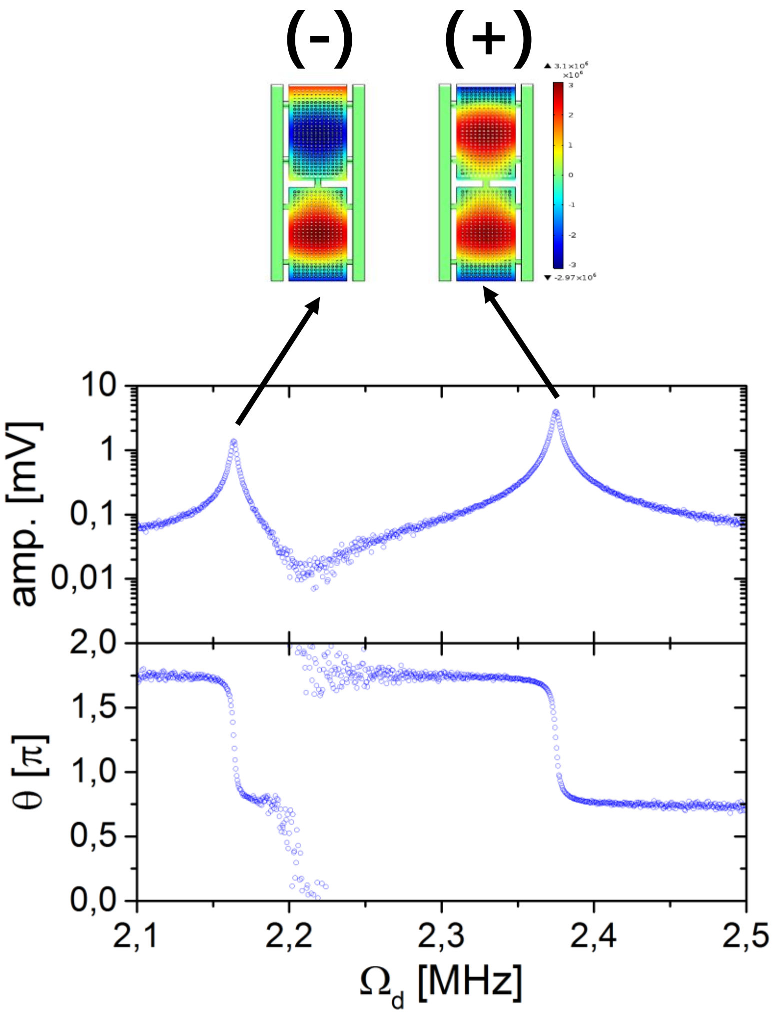
Amplitude **R** and phase **θ** :

Signal quadratures:

$$\mathbf{X = R\cos(\theta)}$$
$$\mathbf{Y = R\sin(\theta)}$$



Linear regime of coupled NEMS



Fundamental mode ~2,2 MHz

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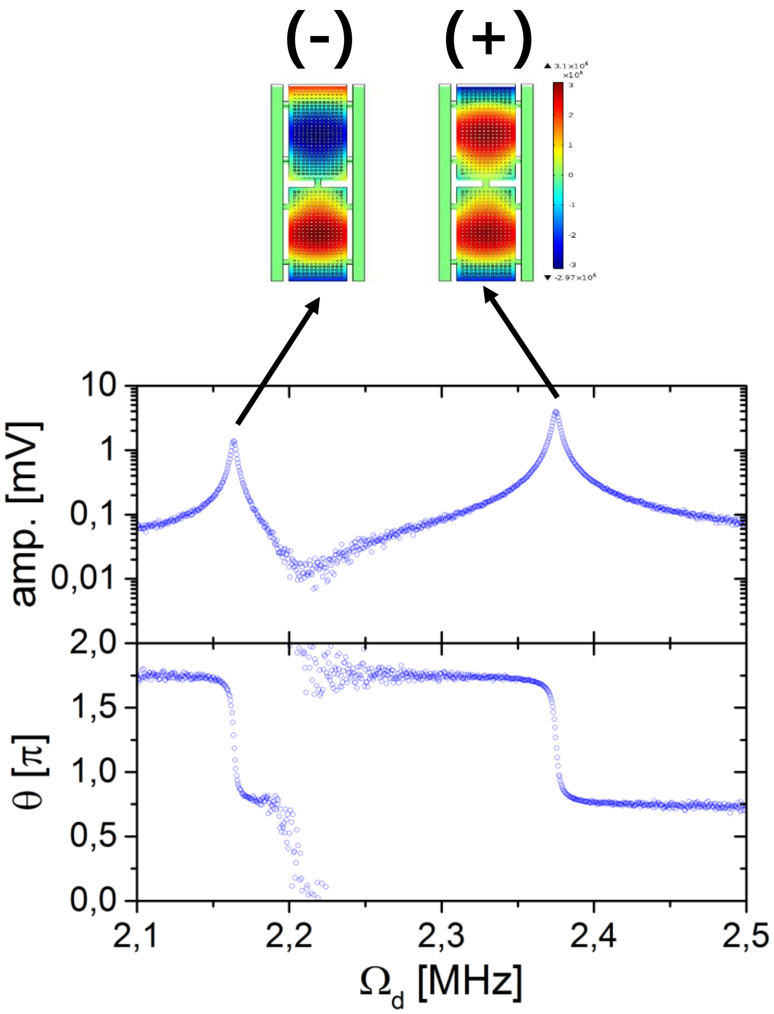

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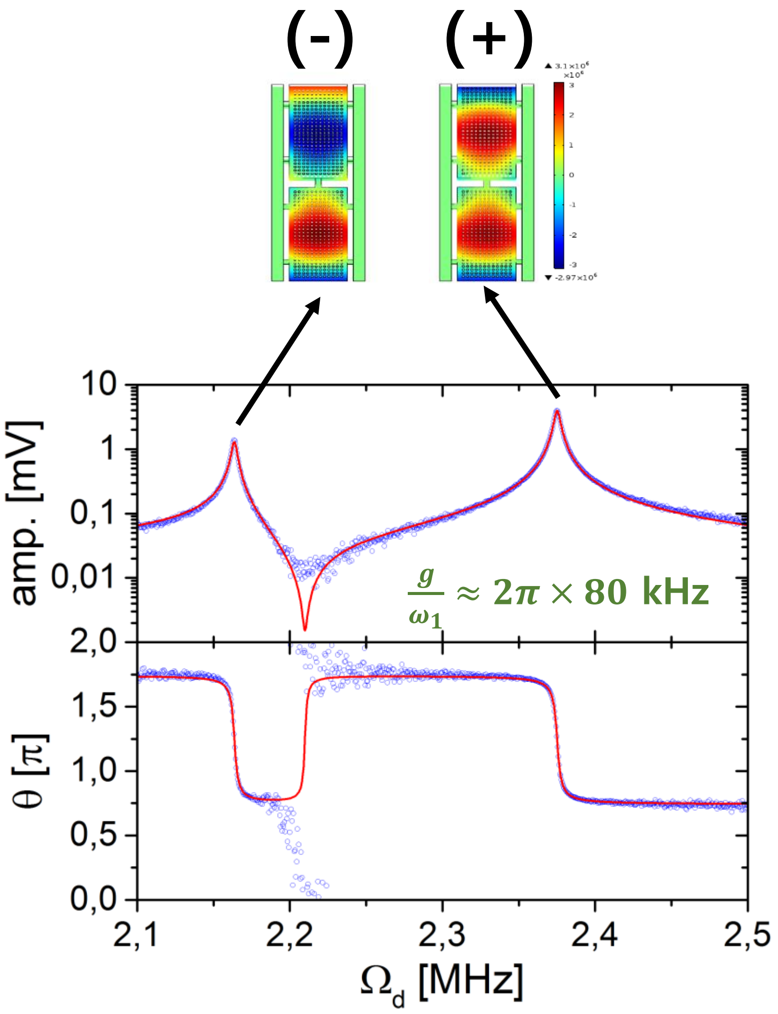

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- **Linear spring coupling**

damping $\Gamma_i \approx 10$ kHz $< \frac{1}{2\pi}\frac{g}{\omega_1}$

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[1] Yong S Joe *et al* 2006 *Phys. Scr.* **74** 259

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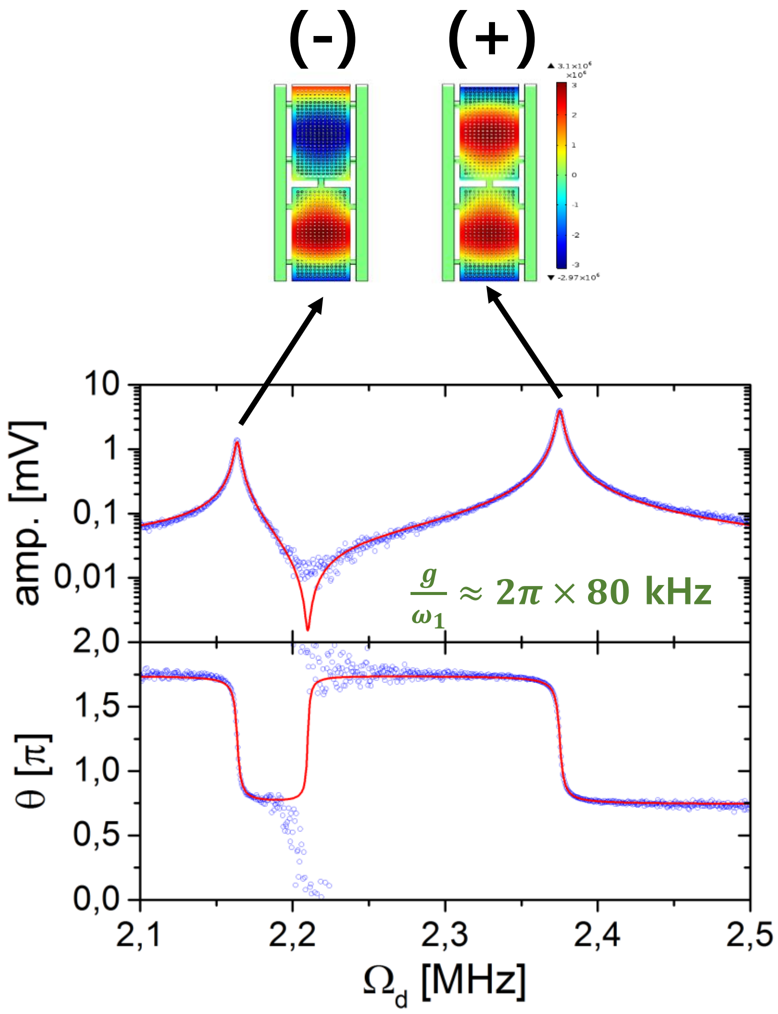

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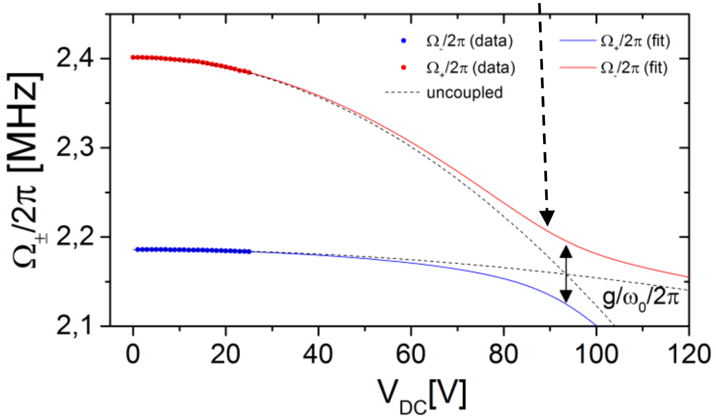
Resonator differentiation :

Static voltage is applied on resonator 2 to shift its natural frequency.

Anticrossing is expected at V_{DC} = 93 V



Mode Ω_- is dominated by membrane 1
Mode Ω_+ is dominated by membrane 2

Same conclusion by thermally shifting membrane with a 840 nm diode laser.

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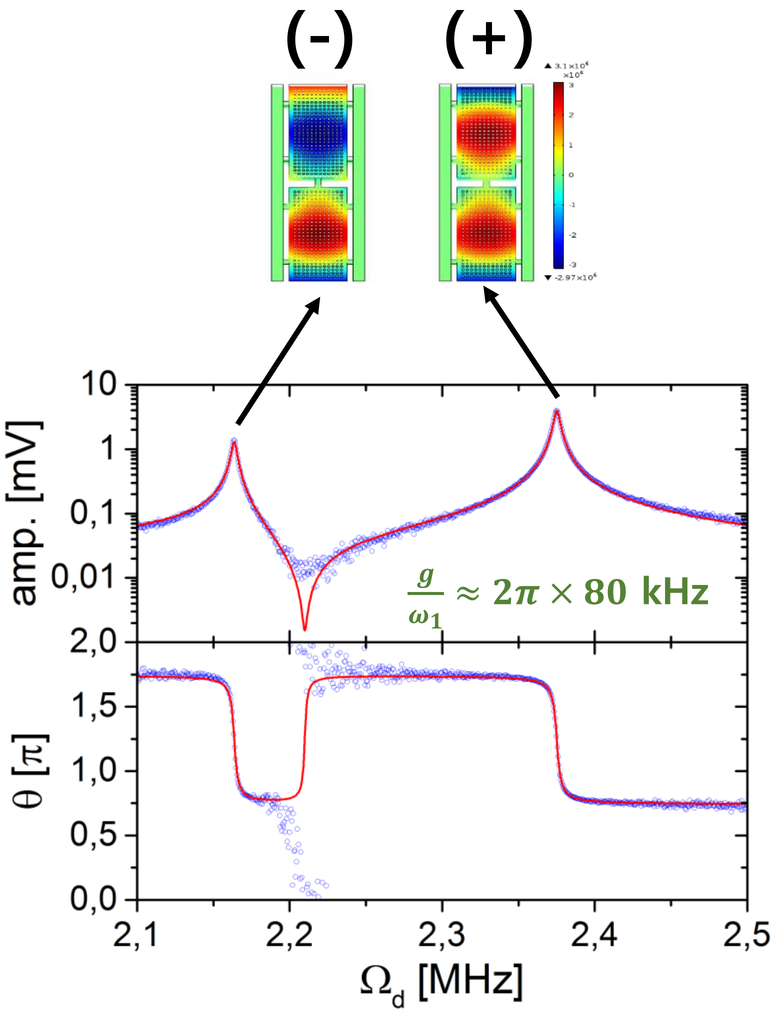

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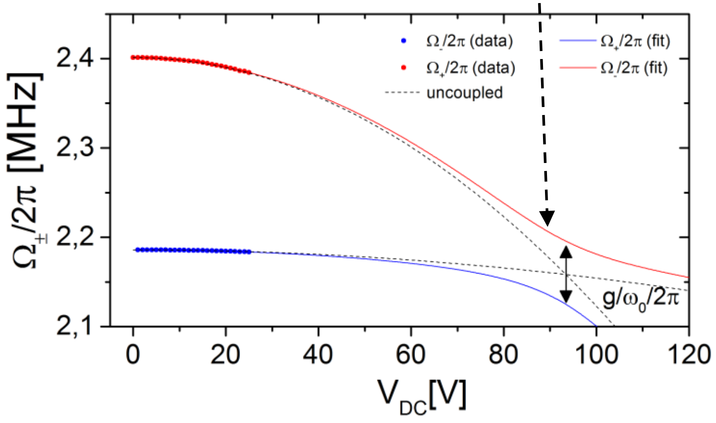

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Full description the system parameters in linear regime

Nonlinear regime

Capacitive force modelization

$F_d = -\frac{dC}{dx} V^2$ with $V = V_{DC} + V_{AC}\cos(\omega_d t)$

- Static force $\propto V_{DC}^2$
- Off-resonant force $\propto V_{AC}^2$
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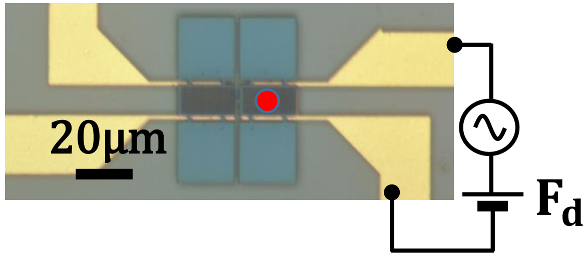

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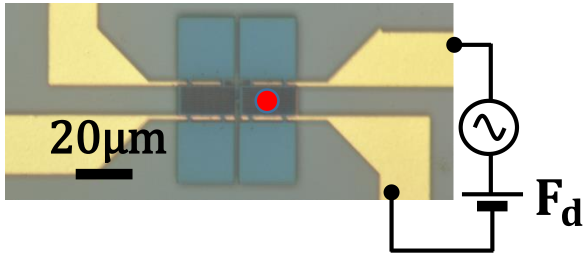

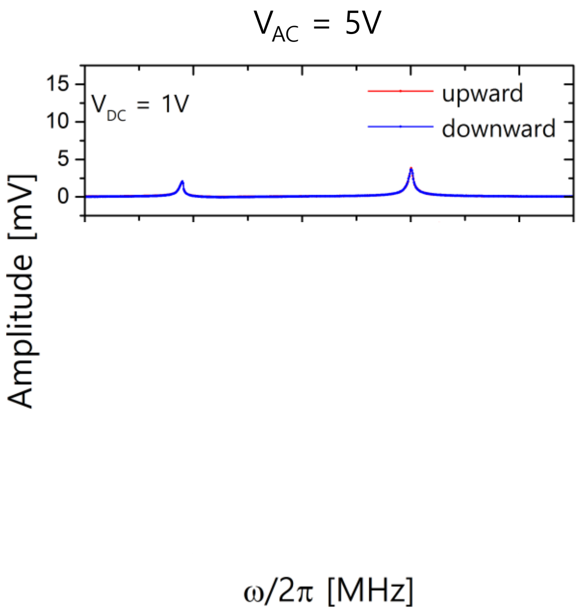

Single Duffing resonator model :

- Anharmonic resonator
- Resonance frequency depends on the amplitude

Coupled Duffing resonators model required

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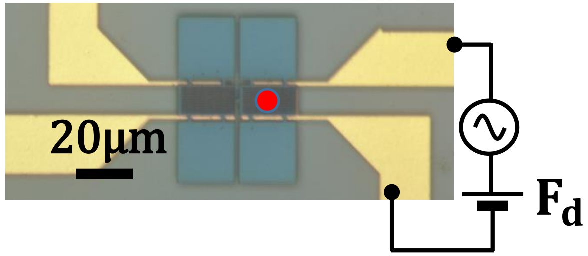

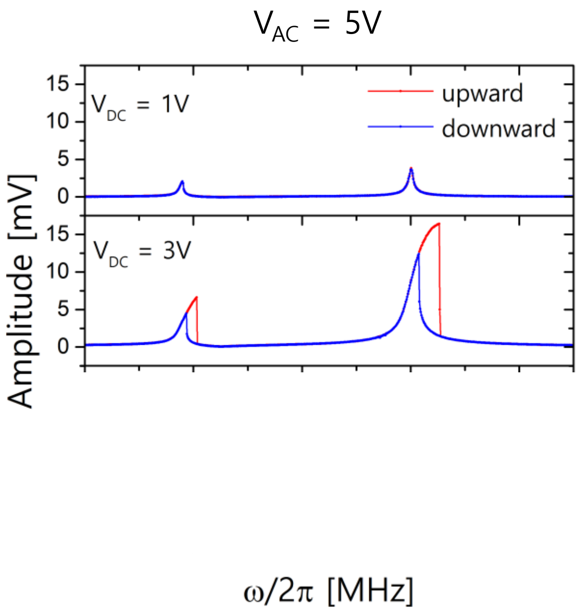

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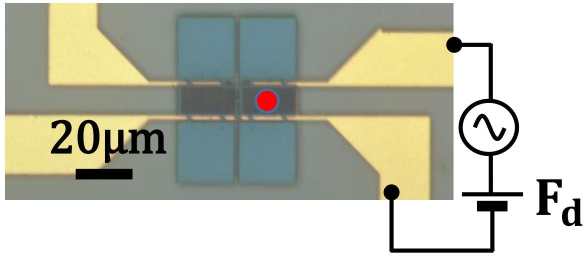

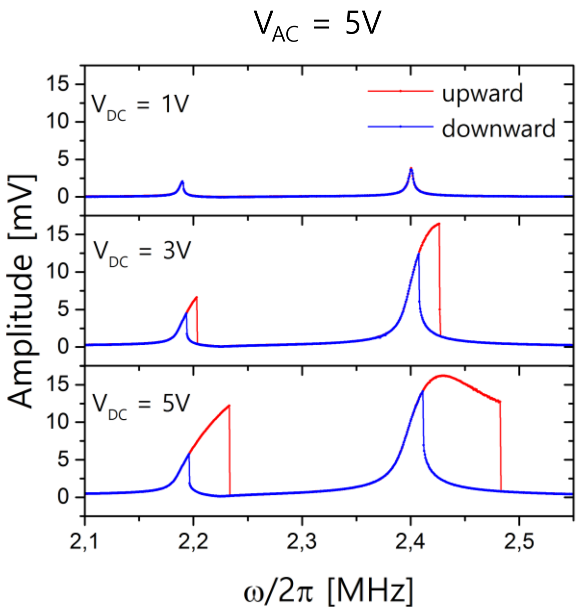

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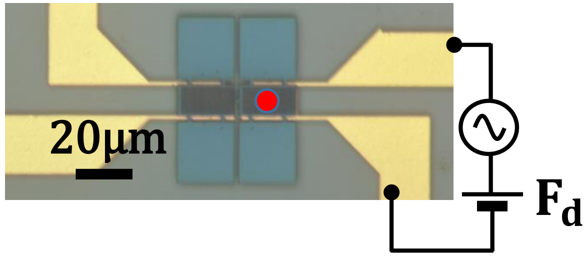

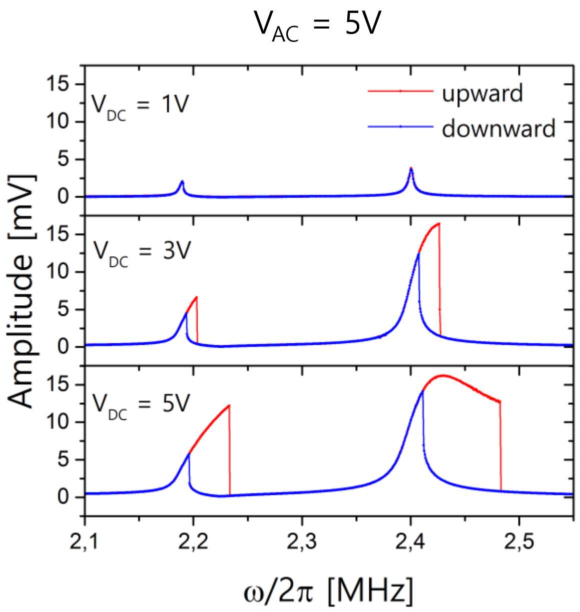

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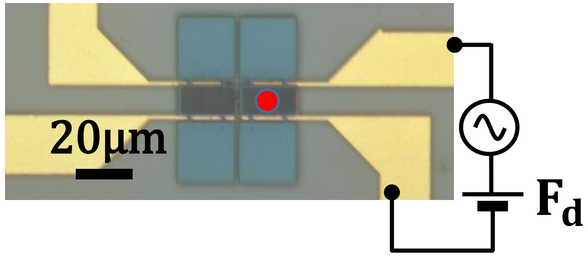

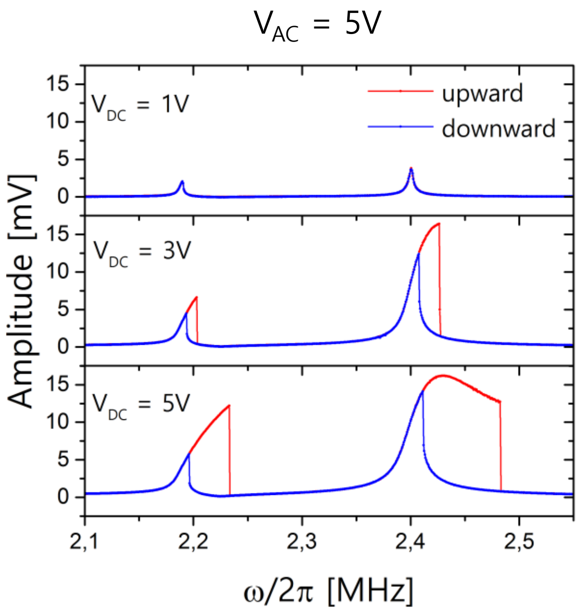

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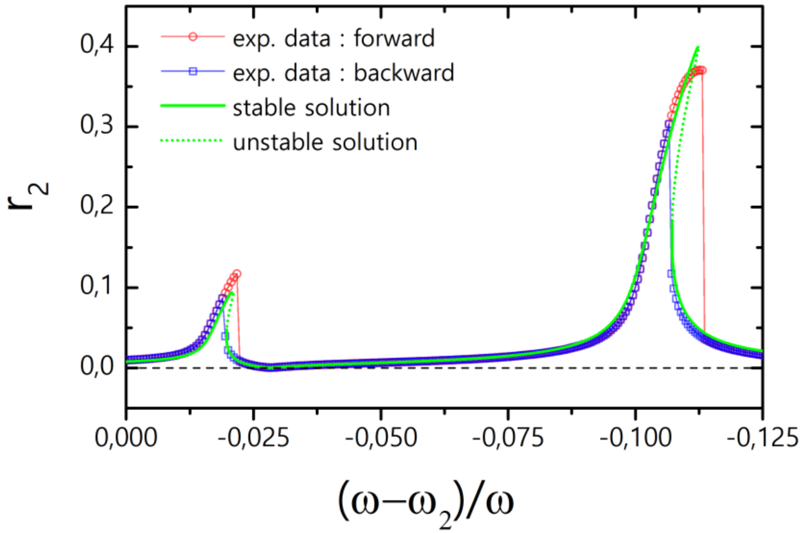


→ nonlinearity $\beta \approx 2\times 10^{-7}\,\mathrm{nm}^{-2}$

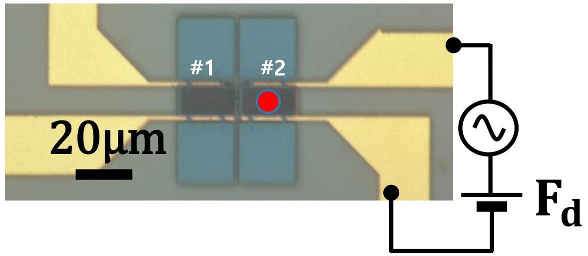
→ applied force $m_{eff}\frac{dC}{dx} \approx 188\,nN$ at 1V

Complete understanding of the non-linear response

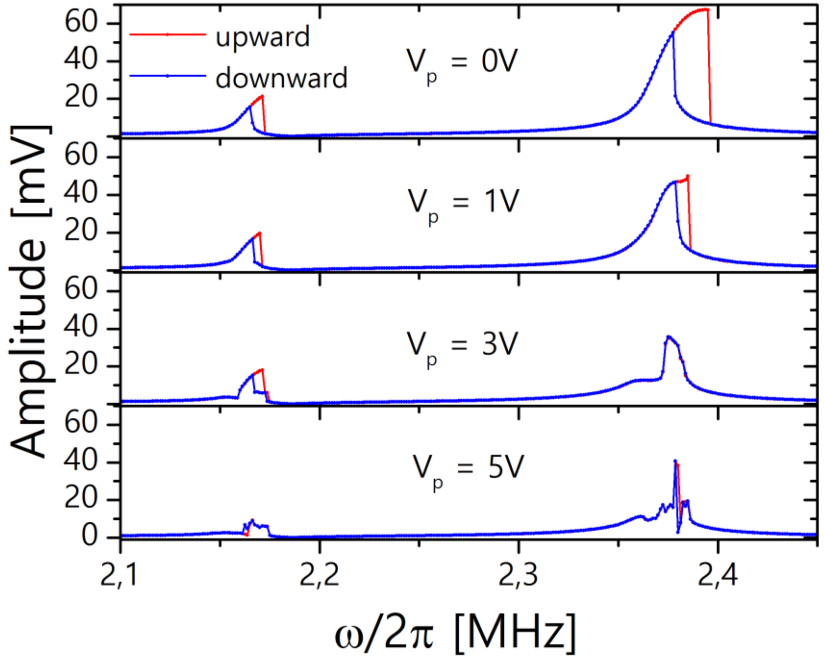
Chaotic amplitude modulation

A slow electrical pump is added :

$$V = V_{DC} + V_{AC}\cos(\omega_d t) + V_p\cos(\omega_p t)$$



$V_{DC} = 2V$; $V_{AC} = 6V$; $\omega_p/2\pi = 8$ kHz



- The membranes are moving at the driving frequency $\omega_d/2\pi$ (carrier)
- The amplitude oscillates at typical frequency $\omega_p/2\pi$

G.Madiot et al., *in progress*

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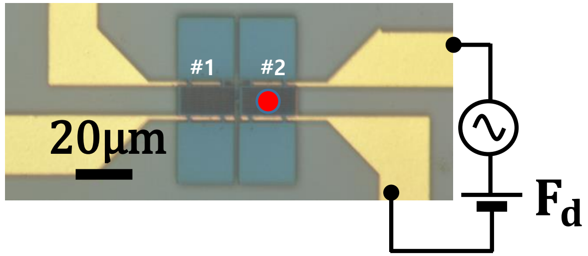

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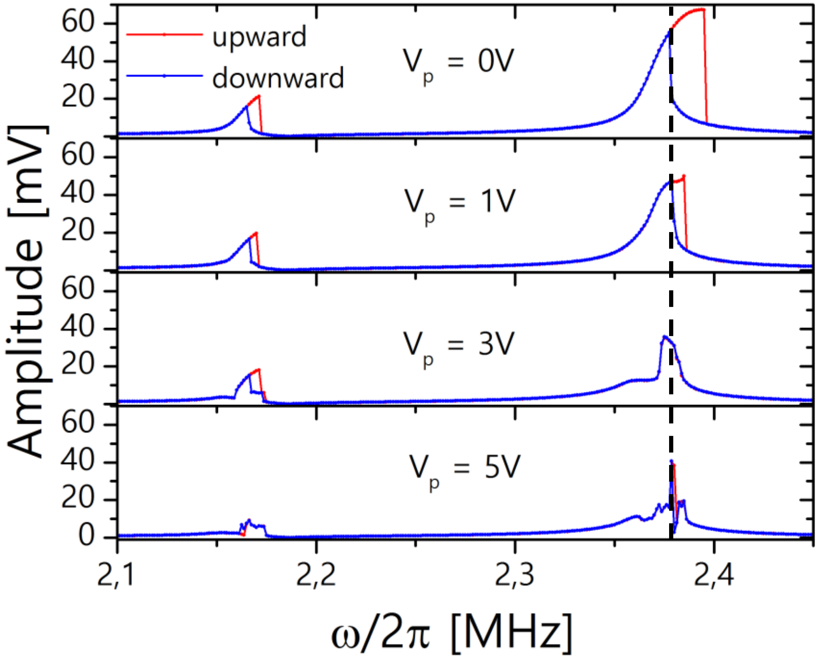

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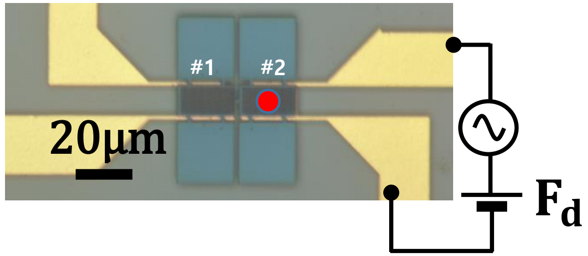

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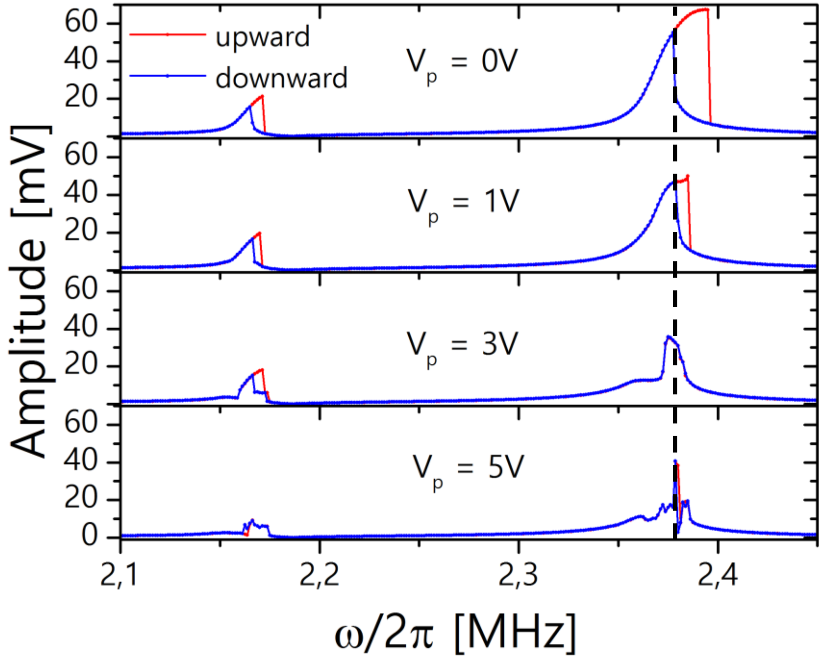
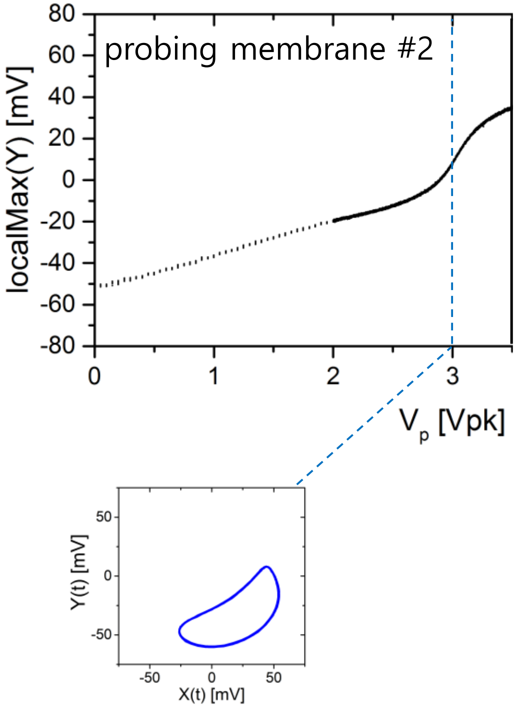


$\omega_d/2\pi$ = 2,379 MHz



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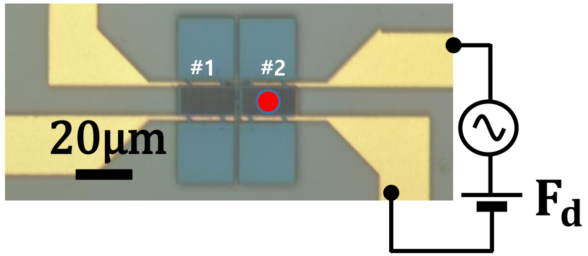

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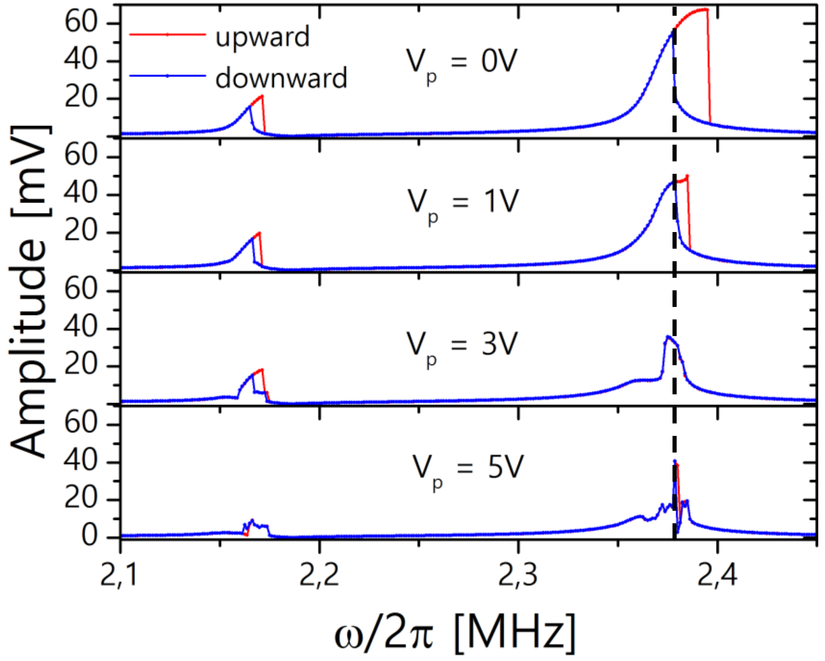

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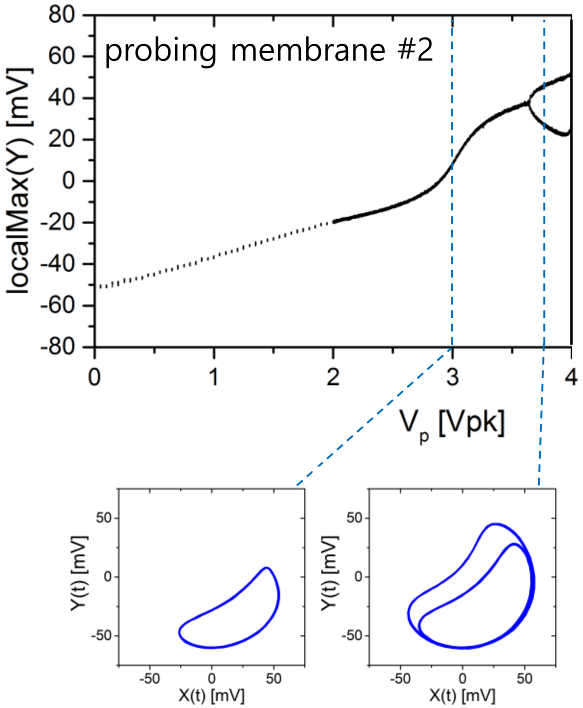


The membranes are moving at the driving frequency $\omega_d/2\pi$ (carrier)

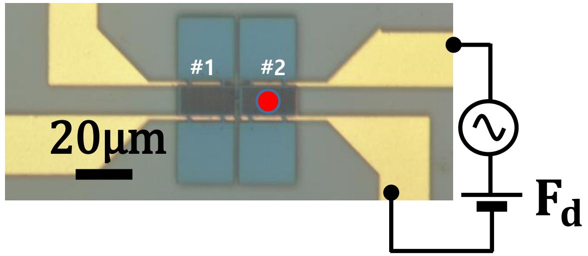
The amplitude oscillates at typical frequency $\omega_p/2\pi$

G.Madiot et al., *in progress*

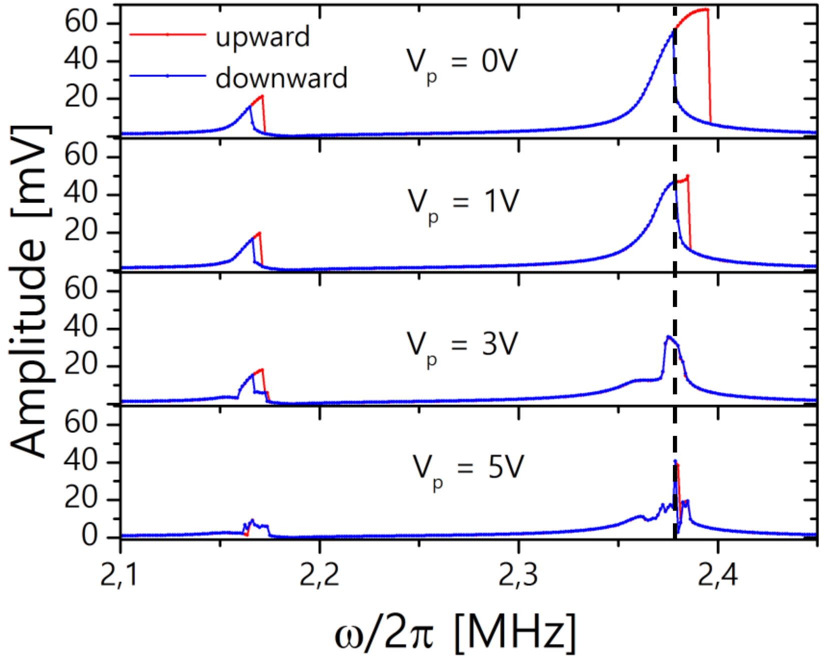
Chaotic amplitude modulation

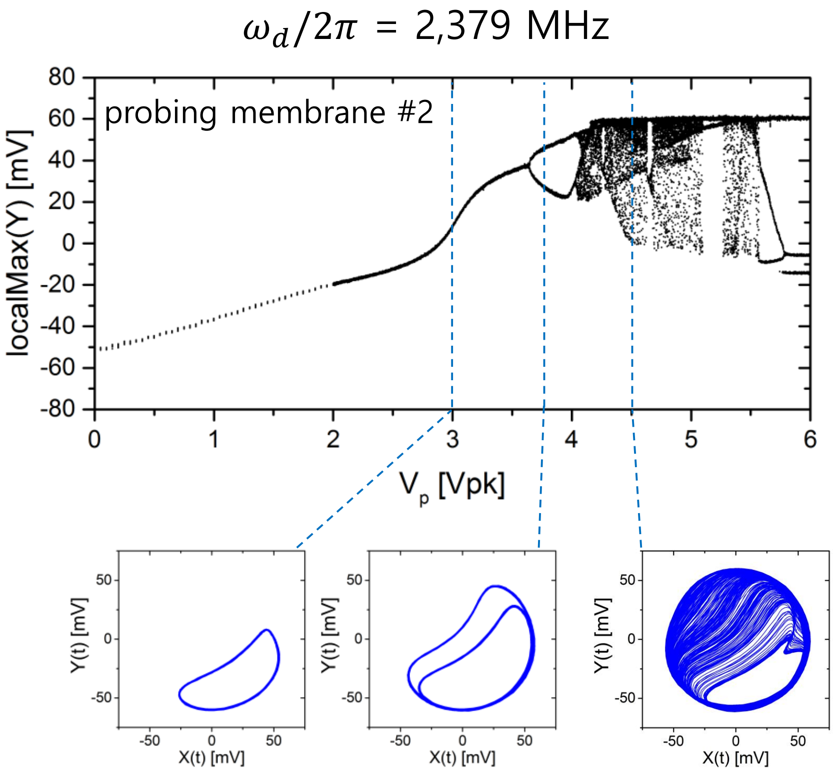
A slow electrical pump is added :

$$V = V_{DC} + V_{AC}\cos(\omega_d t) + V_p\cos(\omega_p t)$$



V_{DC} = 2V; V_{AC} = 6V; $\omega_p/2\pi$ = 8 kHz





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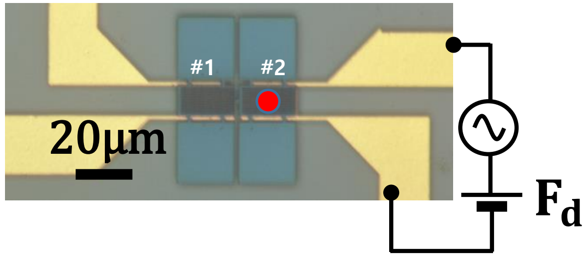

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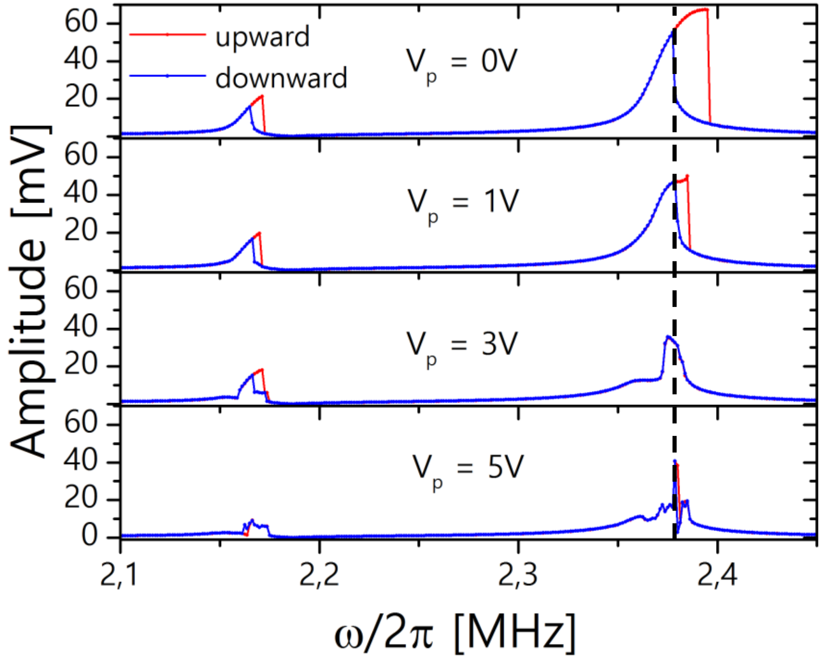

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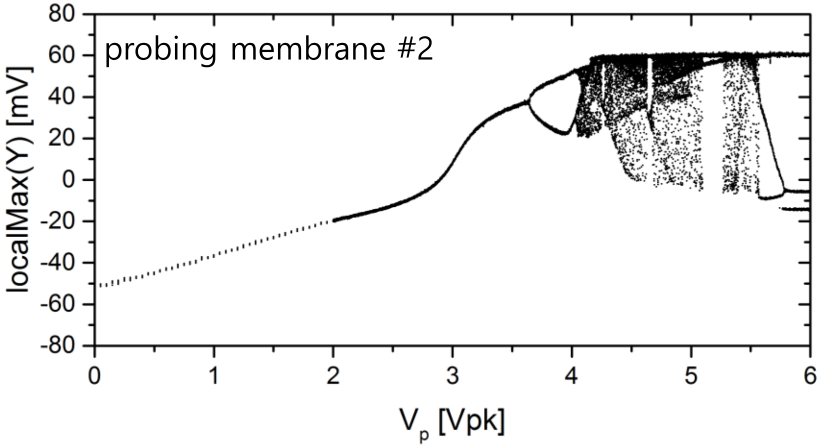

Maximum Lyapunov Exponent (MLE)

Measurement of the exponential growth rate of close trajectories in the dynamical phase space

MLE > 0 → chaotic
MLE ≤ 0 → not chaotic

[2] H. Kantz, *Physics Letters A* **185**, 1 (1994)
[3] R. Hegger et al., *CHAOS* **9**, 413 (1999)

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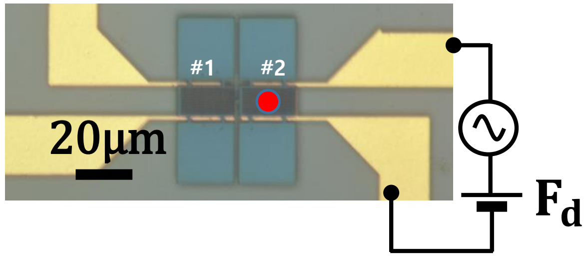

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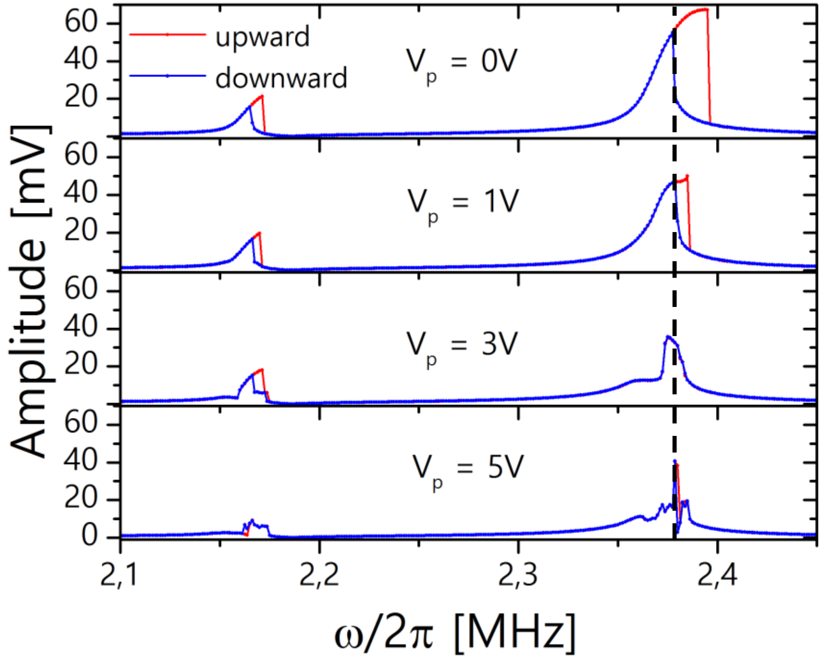

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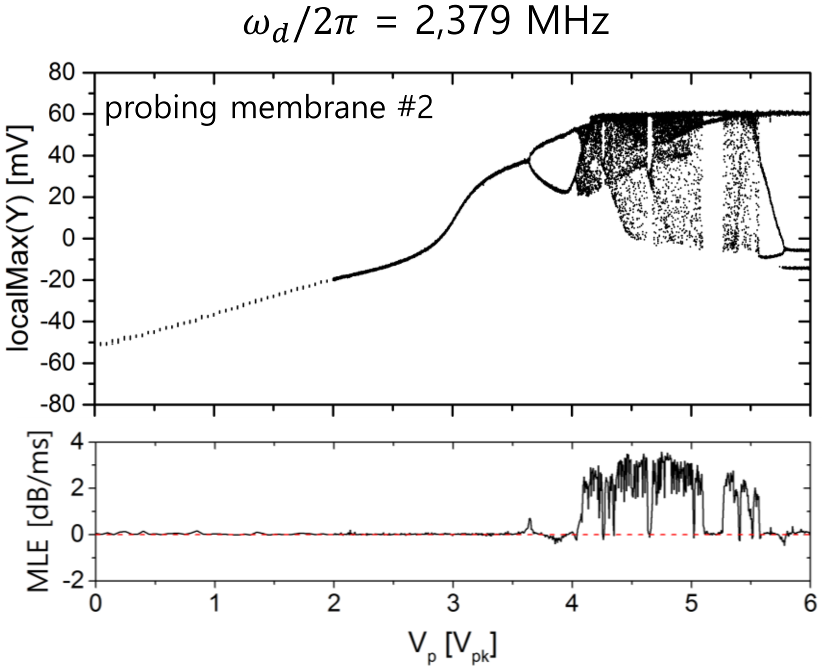

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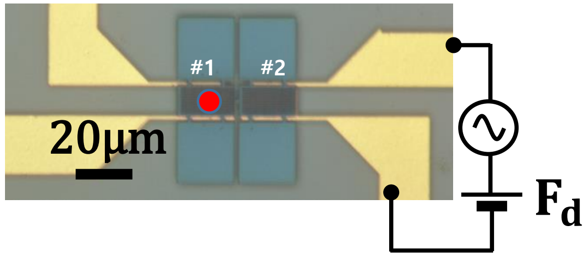

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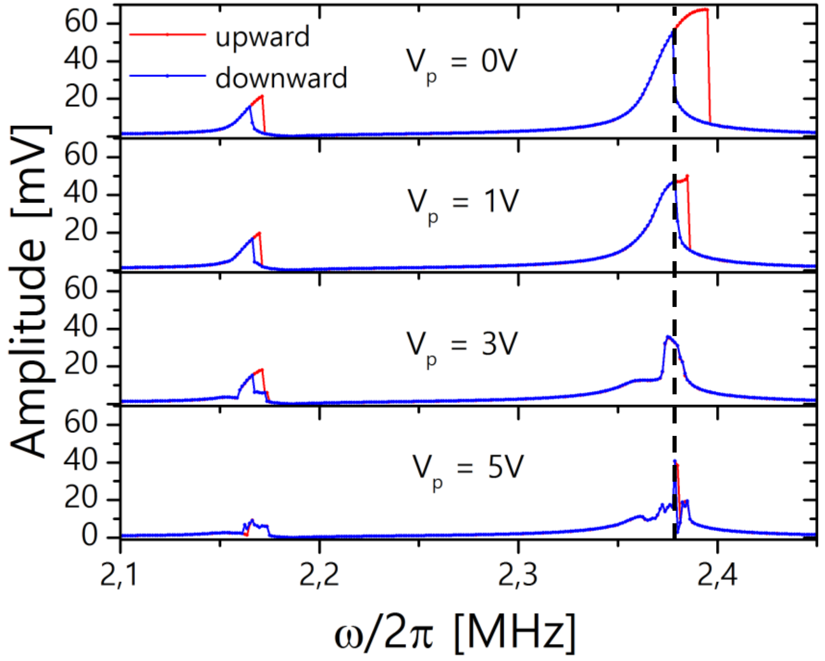

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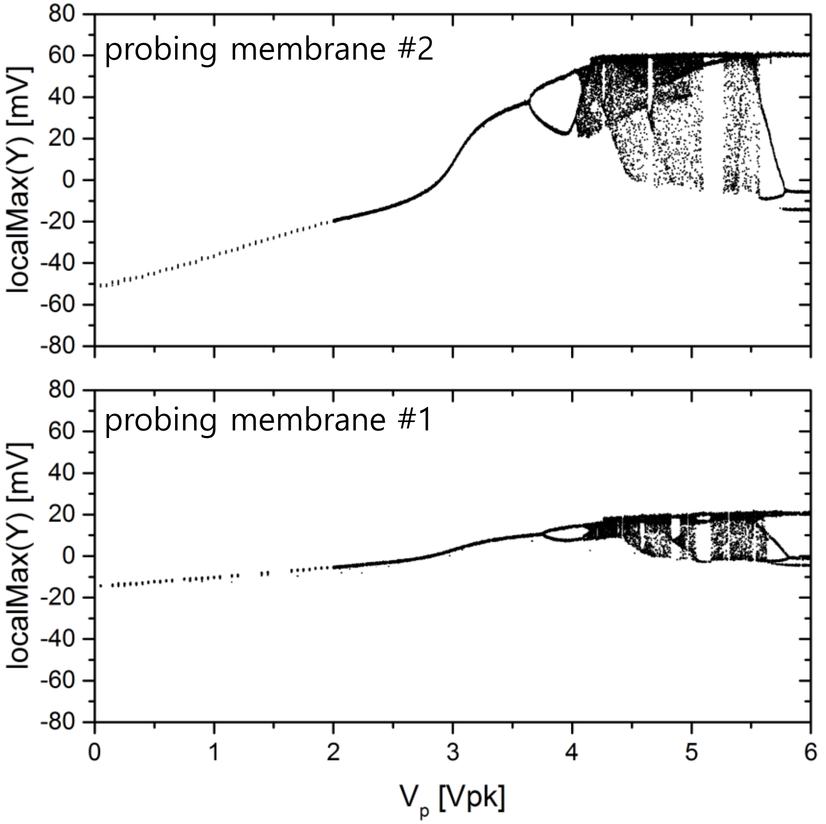

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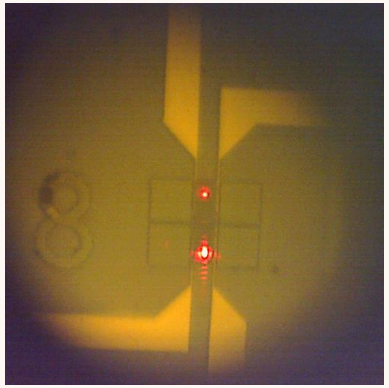
Perspectives

1) Collective dynamics and disorder control

→ Spatial synchronisation of membranes



We can now access the dynamical phase difference between the resonators :

- Collective dynamics (up to 10 coupled nano-membranes)
- Disorder control
- Topological insulators
- …

Perspectives

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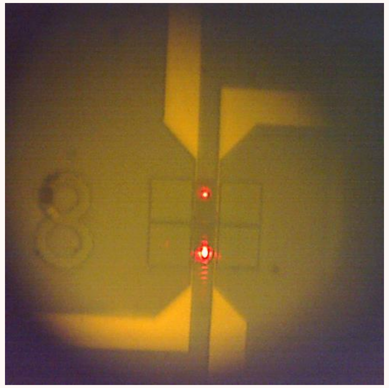

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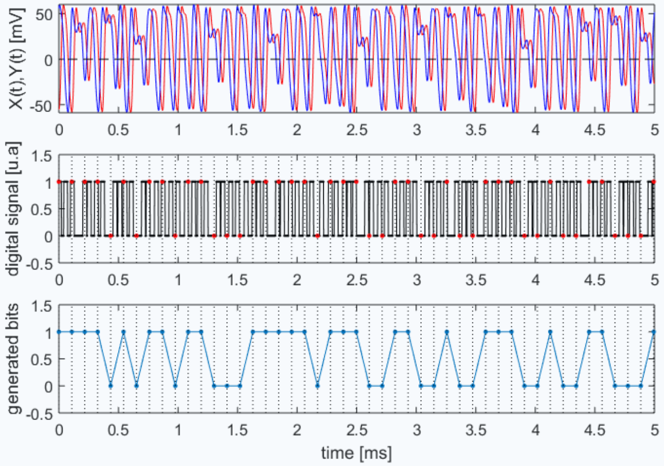
2) Random bit sequences generation



Receipe

- Compare a chaotic time trace $x(t)$ with its delayed-self $x(t+\tau)$
- Do $x(t)$ XOR $x(t+\tau)$
- Sample the obtained bits sequence for a well chosen clock frequency
- Perform statistical tests to check the randomness [6]

[4] M. Sciamanna et al., *Nature Photonics* **9**,151 (2015)
[5] L. Bassham et al., **SP 800-22 Rev. 1a**, NIST (2008)

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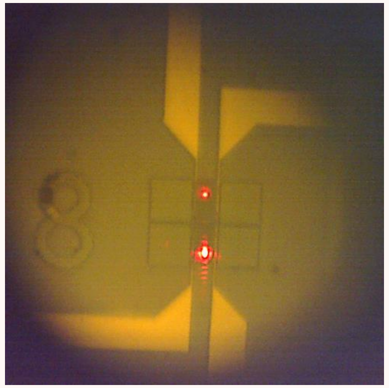

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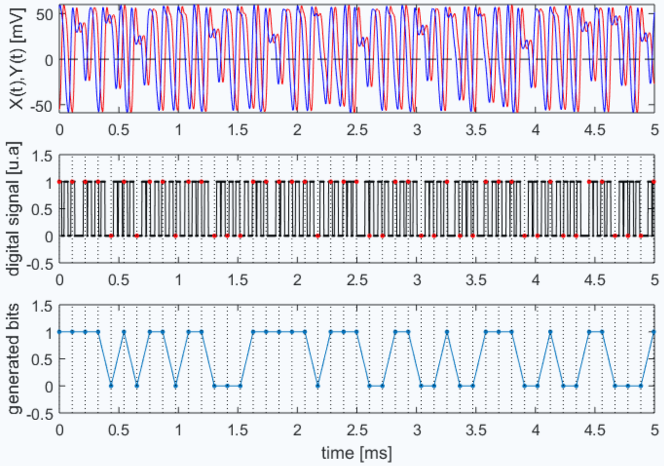

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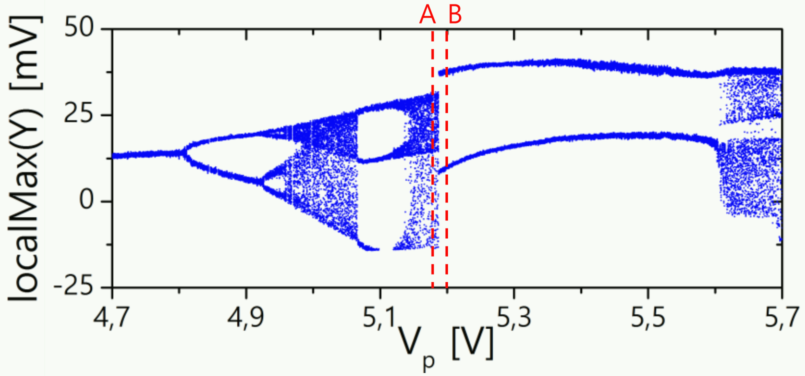
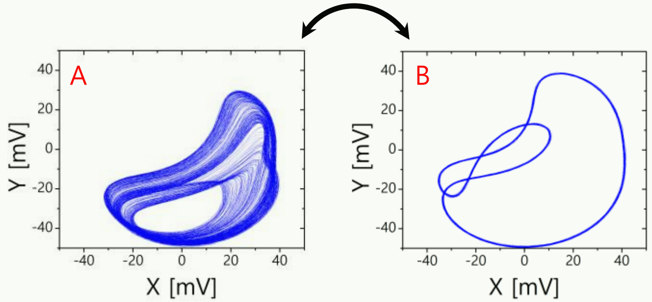
3) Towards chaos-based sensors

→ Exploit a « dramatic crisis » as a sensitive point to an external noise source (heat due to light absorption)



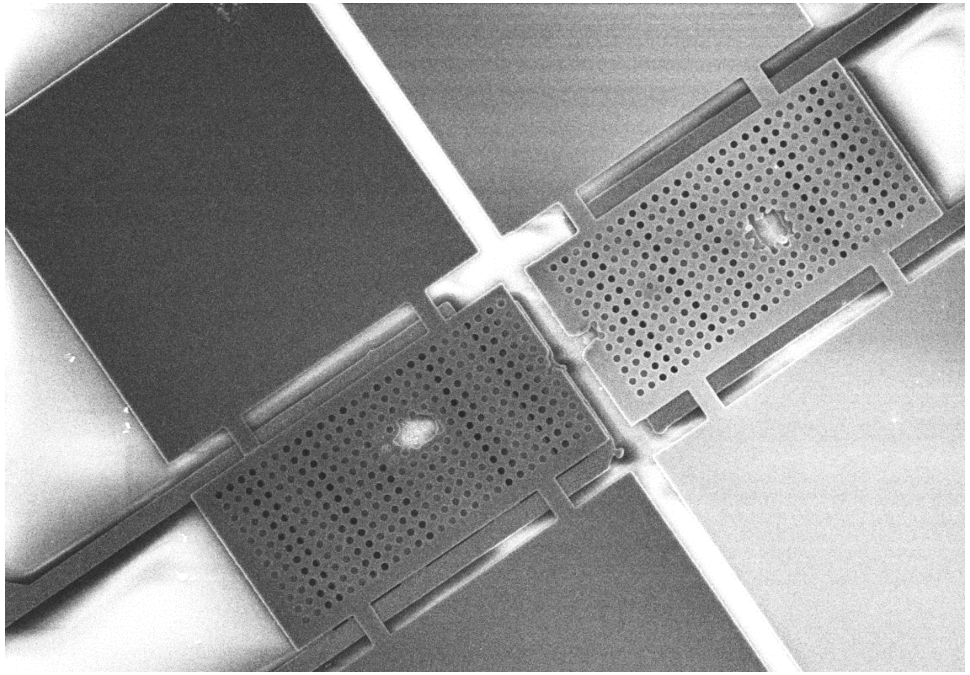
Thermo-optically induced noise



- Mechanical to optical chaos
- Foundation of chaotic systems

[6] J.M.Gambaudo et al., *Nonlinearity*, **1,** 203 (1988)

Thank you !



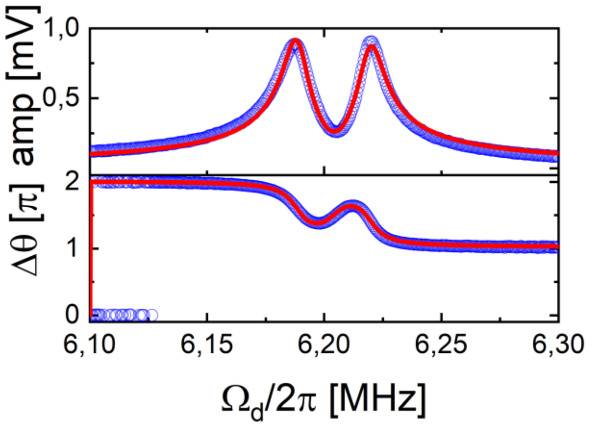
Guilhem Madiot[1]
Franck Correia[1]
Sylvain Barbay[1]
Rémy Braive[1,2]

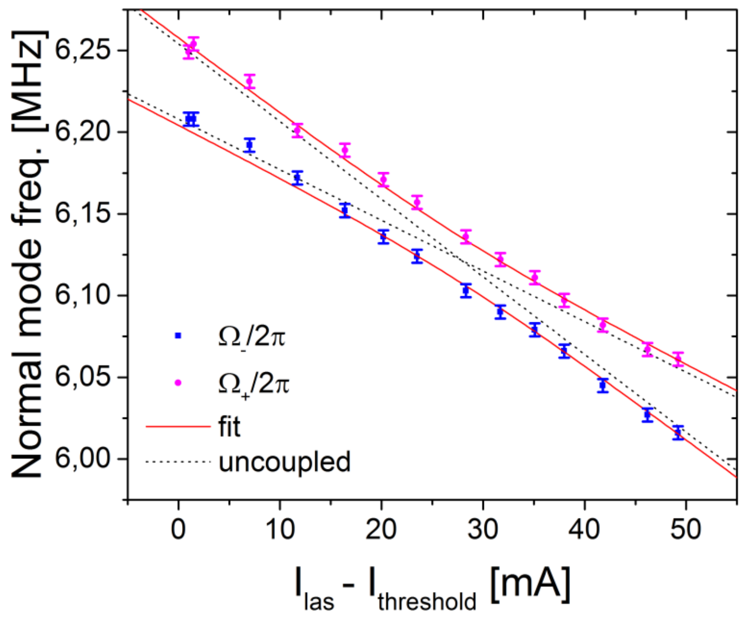
1 : C2N – CNRS, Université Paris-Sud
2 : Universités de Paris

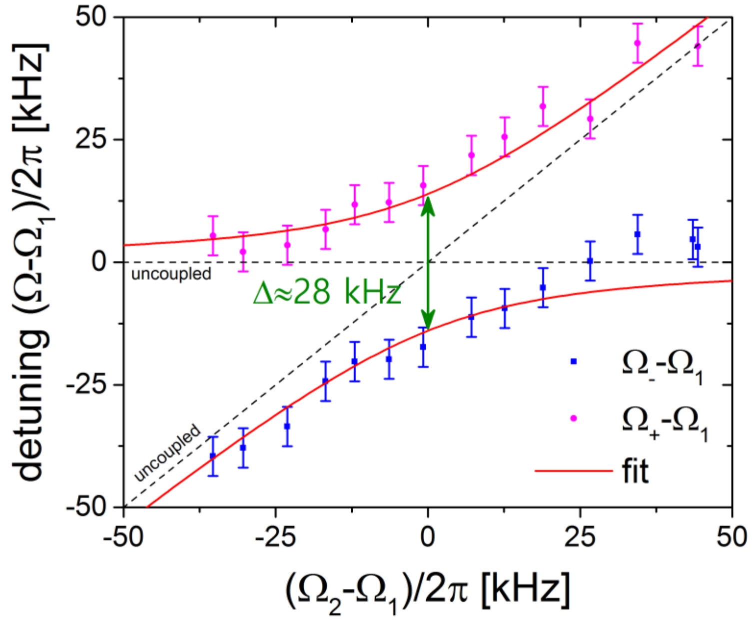
Higher order mechanical modes

g ≈ 6,7 ± 2,1 (rad.MHz)²

$$\Delta \approx \frac{1}{2\pi} g/\Omega_0$$





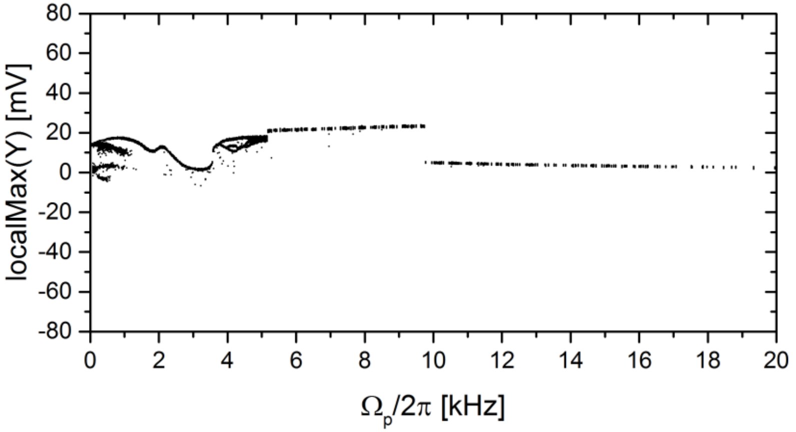


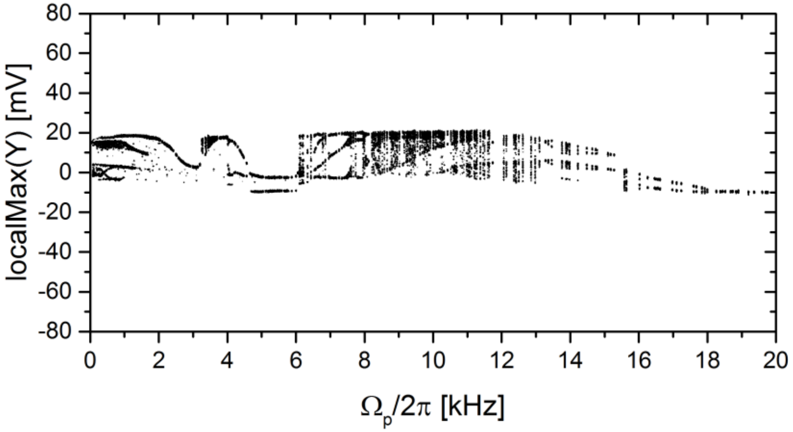
Diagrammes de bifurcations – sweep f_p $V_{AC} = 6\ V_{pk}$ $V_{DC} = 2V$ $V_p = 5V$

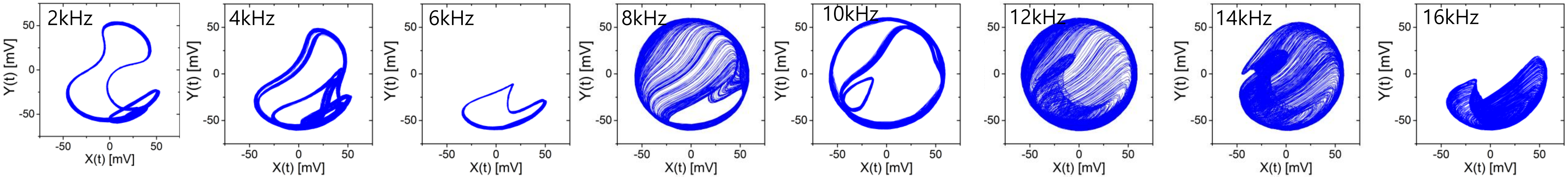
$f_d = 2{,}164$ MHz

$f_d = 2{,}379$ MHz

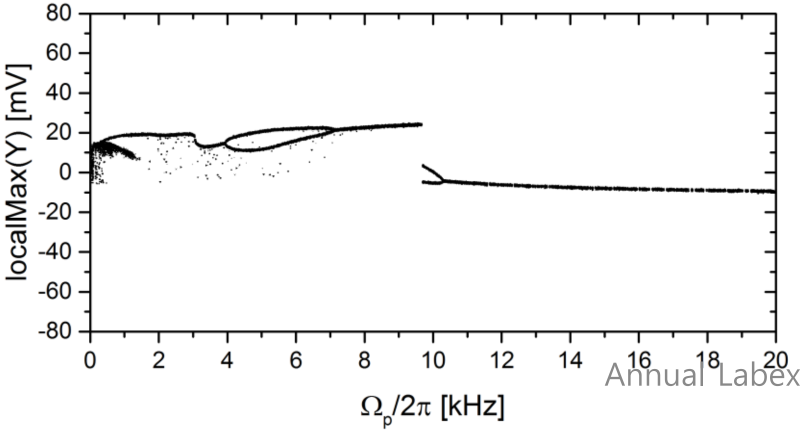
Lect. A

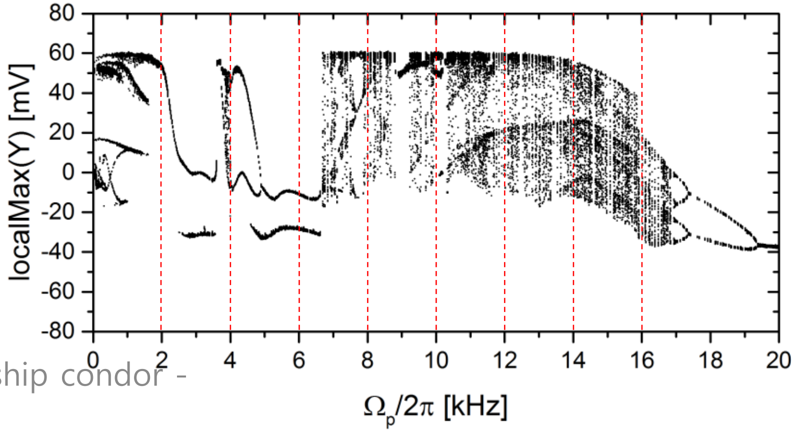






Lect. B

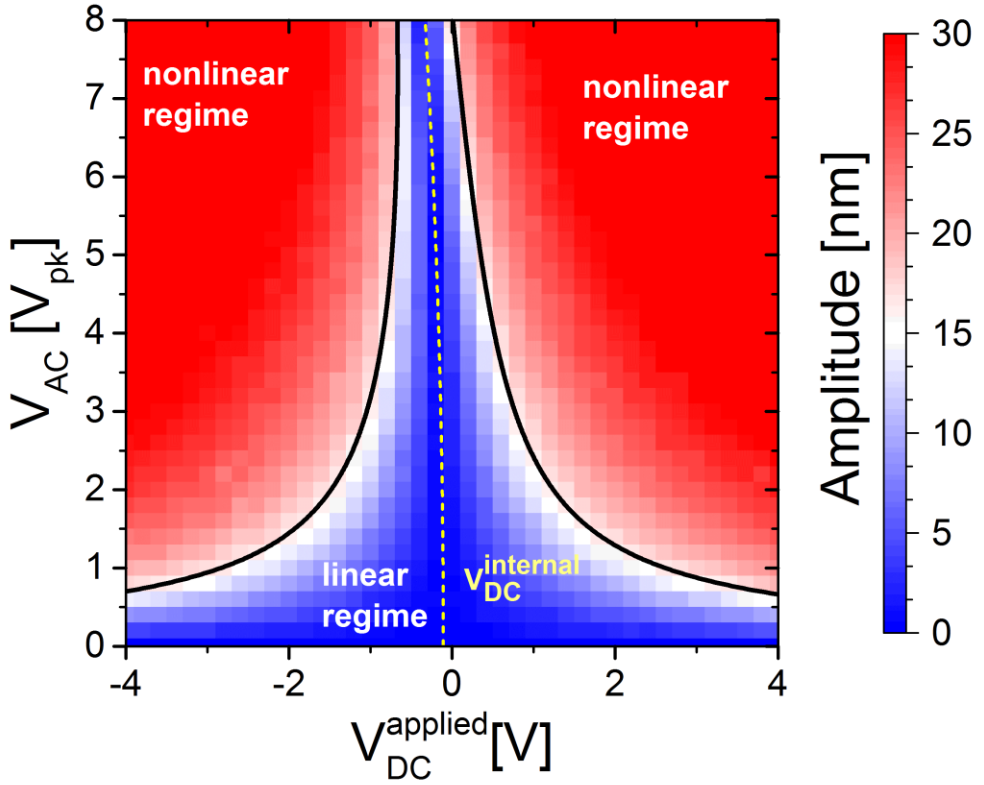




Force modelization check

$\mathbf{Applied\ Force} \propto \boldsymbol{V_{DC}V_{AC}}$ with $\mathrm{V_{DC}} = \mathrm{V_{DC}^{internal}} + \mathrm{V_{DC}^{applied}}$

Amplitude of ω_+ mode
in the $\{V_{AC}, V_{DC}\}$ space

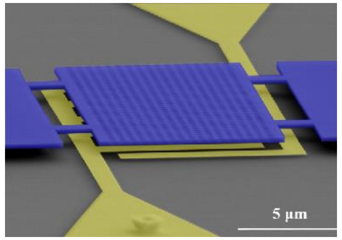


- In linear regime, one can verify $A_\omega \propto V_{AC}V_{DC}$
- Duffing regime begins at threshold F_{thre}
- It implies a threshold line $V_{AC} \propto \frac{F_{thre}}{V_{DC}}$ in the $\{V_{AC}, V_{DC}\}$ space
- Note that $V_{DC}^{applied}$ slightly varies with V_{AC}

Perspectives

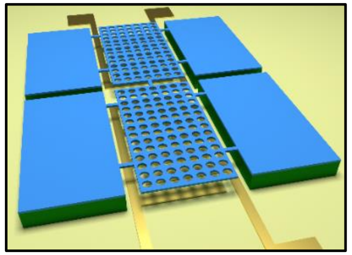
The new generation of sample is already there !

1 resonator



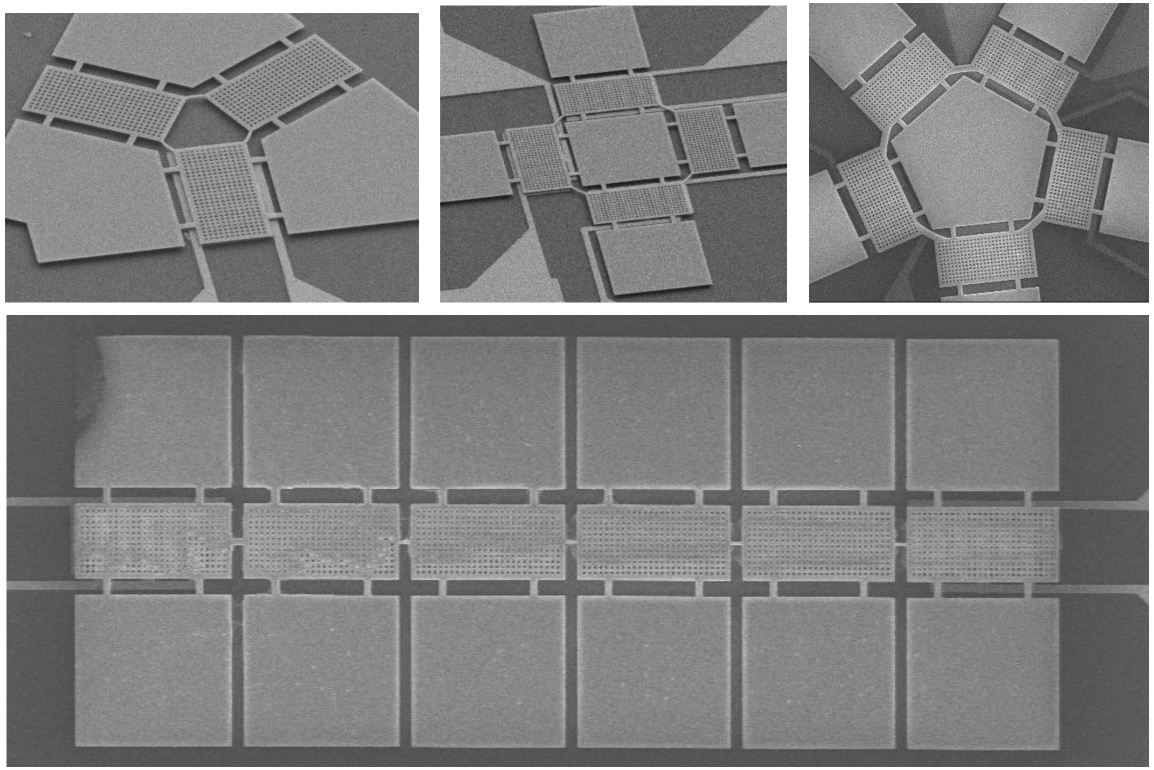
2013-2016

2 coupled resonators



2016-2020

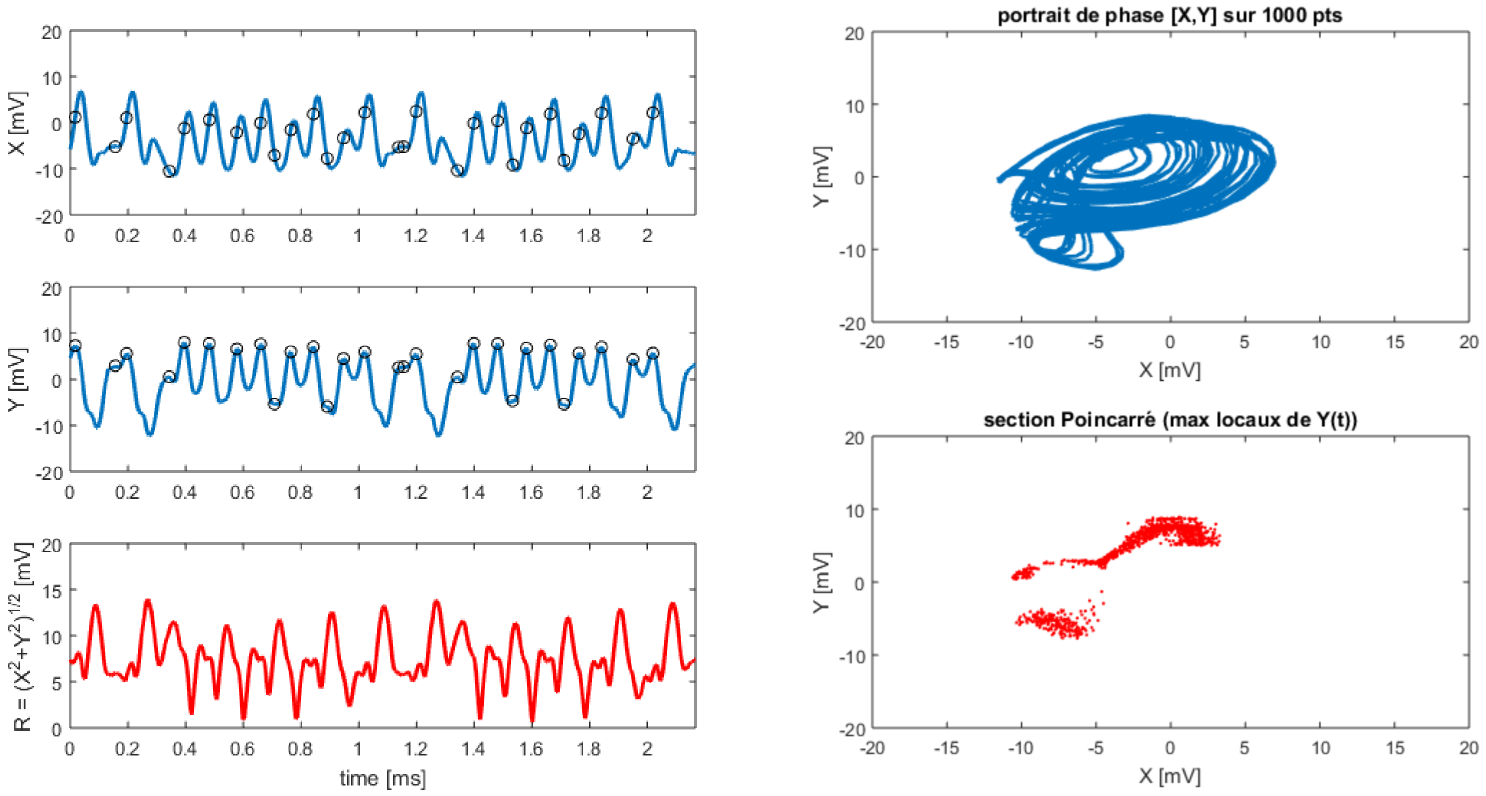
1D-Arrays of coupled resonators



2020 - ?

- Collective dynamics
- Disorder control
- Topological insulators
- …

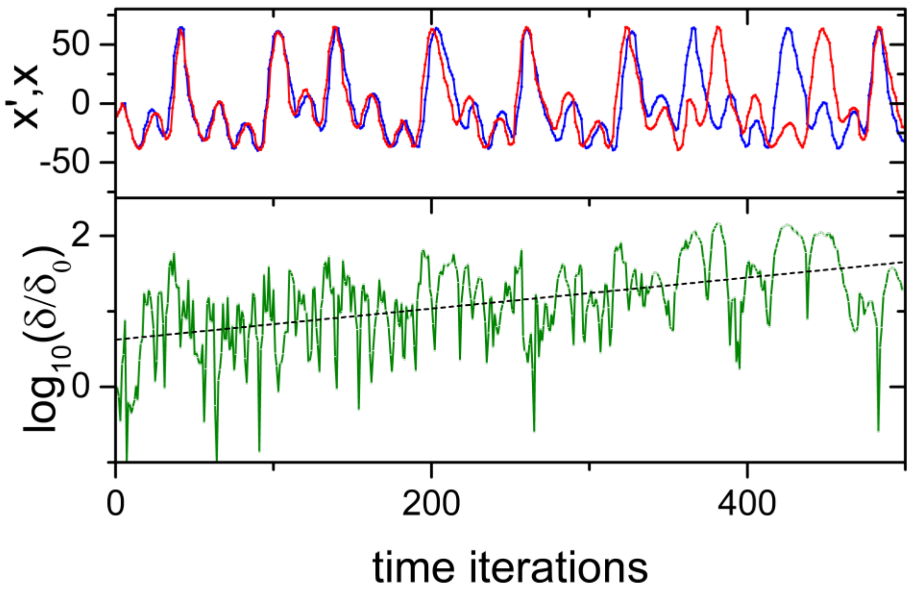
How to build a Poincarré section



How to evaluate a Lyapunov Exponent

Method

- Reconstruc the d-dimensionnal phase space using delay-embedding
- Find close neighbors of the trajectory at given instant t_0
- Check error function evolution with time
- Determine convergence rate (⇔Lyapunov exponent) with linear fit (in log scale)



Get close neighbors

- Take all points contained in the d-sphere of radius ε (arbitrarily chosen) around the initial point
- The d-sphere evolves in a d-ellipsoid following a expansion rate in the i^{th} dimension given by $e^{\lambda_i t}$

Note

Lyapunov spectrum $\{\lambda_1 > \lambda_2 > \ldots > \lambda_d\}$
λ_i is the divergence rate of the system in the i^{th} dimension.

Only the Maximum Lyapunov Exponent (MLE) is calculated here.